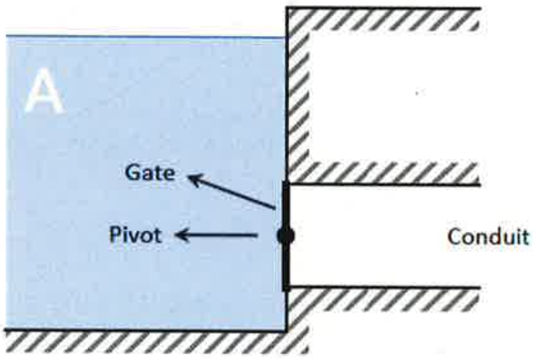
> **NOTE:** In multiple-choice questions, choose **ONLY** one answer.

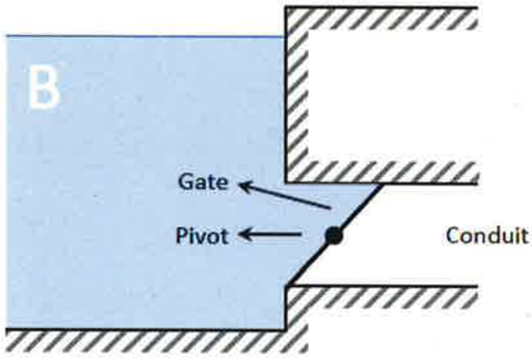
Questions:

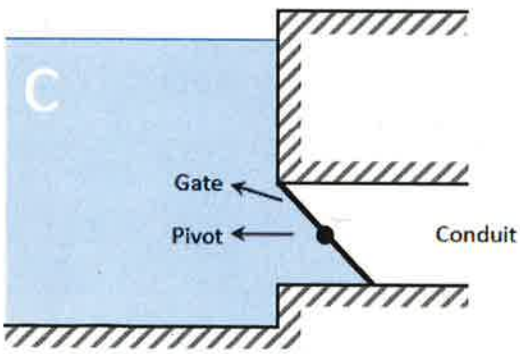
1. Consider the following three cases sketched below in which there is a gate that controls the water flow from a water dam to a conduit. The gates can pivot around the points marked in the illustrations.

If no force or torque is applied to the gate, please detail for each case:

- Will the gate open by the action of the water pressure or not?
- In case it opens, in which sense will the gate rotate?
    - Clockwise ↻
    - Counterclockwise ↺







A: Open, counterclockwise
B: Close
C: Open, counterclockwise

Provide a short explanation that justifies your reasoning.

D

2. A velocity vector $\vec{V}$ in two-dimensional flow is inclined at an angle θ to the x-axis. The resulting acceleration vector $\vec{a}$

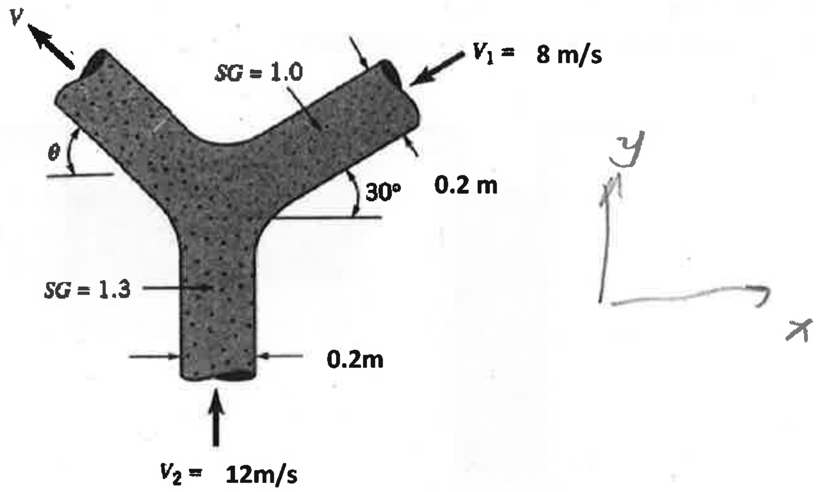
(a) will be always normal to $\vec{V}$

(b) will be always parallel to $\vec{V}$

(c) will have an inclination of (90-θ) to the y-axis

(d) will have an inclination α to the x-axis which depends on the components of the acceleration.

3. Two jets of liquid, one with specific gravity 1.0 and the other with specific gravity 1.3, collide and form one homogenous jet as shown below. Determine the speed, V, and the direction θ, of the combined jet. Gravity is negligible.



Continuity eq.:

$$\gamma V A = \gamma_1 V_1 A_1 + \gamma_2 V_2 A_2$$

$$\frac{\gamma V \pi d^2}{4} = \gamma_1 V_1 \frac{\pi d_1^2}{4} + \gamma_2 V_2 \frac{\pi d_2^2}{4}$$

$$\gamma V d^2 = \gamma_1 V_1 d_1^2 + \gamma_2 V_2 d_2^2 = 1 \times 8 \times 0.2^2 + 1.3 \times 12 \times 0.2^2 = 0.944$$

momentum eq. in y direction:

$$\gamma V^2 d^2 \sin\theta = \gamma_2 V_2^2 d_2^2 - \gamma_1 V_1^2 d_1^2 \sin\frac{\pi}{6} = 6.208$$

$$V \sin\theta = \frac{6.208}{0.944} = 6.58 \text{ m/s} \quad \text{---} \quad ①$$

momentum eq. in x direction

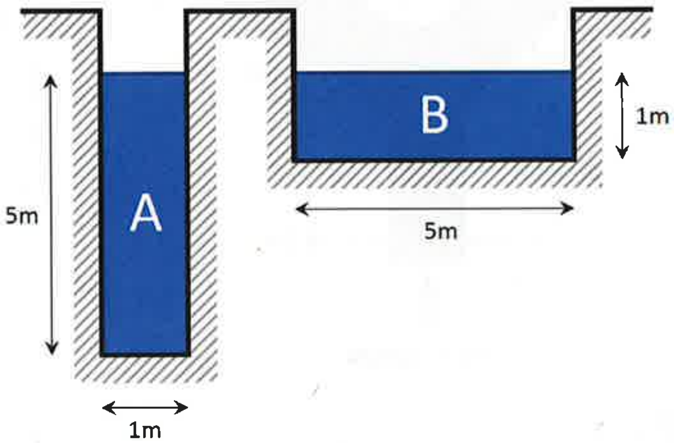
$$\gamma V^2 d^2 \cos\theta = \gamma_1 V_1^2 d_1^2 \cos\frac{\pi}{6} = 2.22$$

$$V \cos\theta = \frac{2.22}{0.944} = 2.35 \text{ m/s} \quad \text{---} \quad ②$$

$$V = \sqrt{①^2 + ②^2} \approx 7 \text{ m/s} \qquad \theta = \arctan\left(\frac{①}{②}\right) \approx 70°$$

4. Consider uniform flow of water in the two channels shown. They both have the same slope, the same wall roughness. Please choose one of the options and justify it:

- $Q_A < Q_B$
- $Q_A = Q_B$
- $Q_A > Q_B$
- There is not enough data to choose one



Note: it is not necessary to calculate any numbers, but a formula is compulsory to prove you choice.

Area : $A_A = A_B$

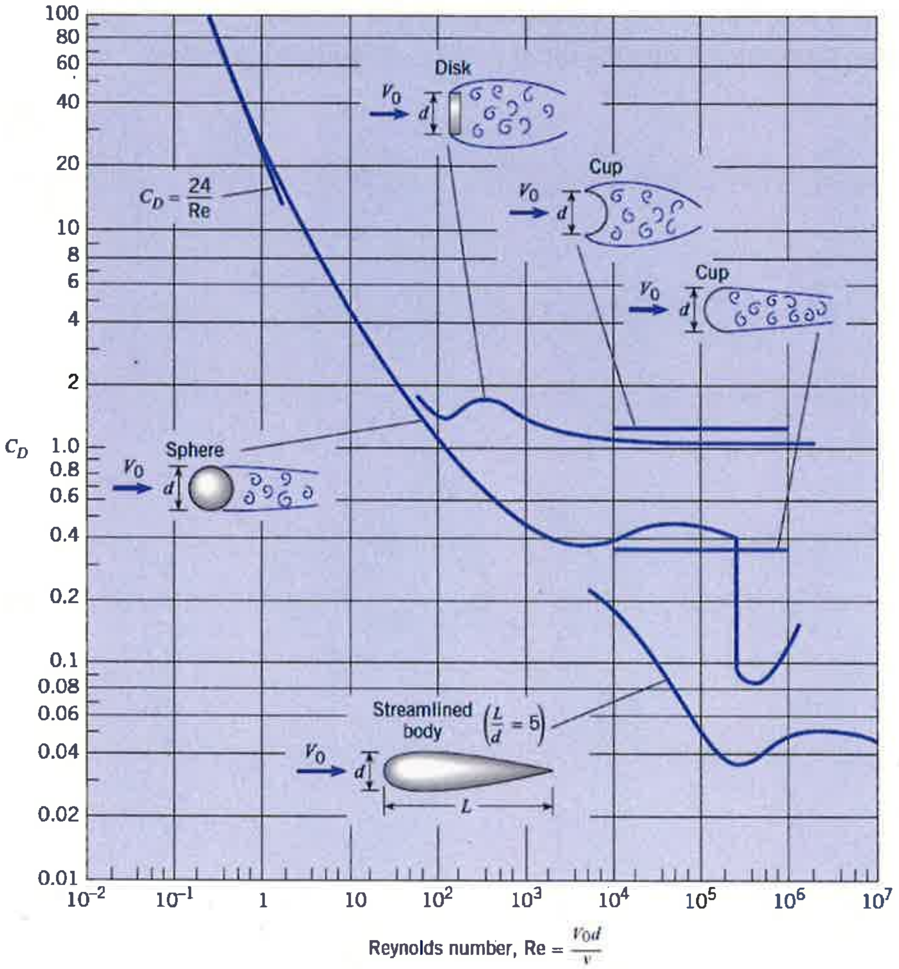
perimeter : $P_A = 11\,m > P_B = 7\,m$

Manning's eq. $Q = \frac{1.49}{n} A R_h^{2/3} S_o^{1/2}$

$R_h = \frac{A}{P}$

$R_{h-A} < R_{h-B} \Rightarrow Q_A < Q_B$

5. Consider a smooth sphere with diameter 20cm. Calculate the drag force when moving in air at a velocity of 27 km/h and 75km/h. Briefly comment on the difference between the resulting forces and the velocities.



Note: use $\nu = 1.5 \cdot 10^{-5} m^2/s$ and $\rho_{air} = 1\ kg/m^3$ to simplify your calculations.

$$Re_{27\frac{km}{h}} = \frac{27/3.6 \times 0.2}{1.5 \times 10^{-5}} \approx 1 \times 10^5 \quad C_{D_{27}} = 0.5$$

$$Re_{75\frac{km}{h}} = \frac{75/3.6 \times 0.2}{1.5 \times 10^{-5}} = 2.78 \times 10^5 ; \quad C_{D_{75}} = 0.085$$

$D_{27} = 0.44N$ $\qquad D_{75} = 0.58N$

Problems:

1. A thin plate having a width w and a height h is located so that it is normal to a moving stream of fluid. Assume the drag force D, that the fluid exerts on the plate is a function of w and h, the fluid viscosity μ and density ρ, respectively, and the velocity V of the fluid approaching the plate. Determine a suitable set of π terms to study this problem experimentally.

$[w] = L$, $[h] = L$, $[D] = \frac{ML}{T^2}$, $[\rho] = \frac{M}{L^3}$, $[\mu] = \frac{M}{LT}$, $[V] = \frac{L}{T}$

Number of π group is $6 - 3 = 3$

Let $D = w^a h^b \mu^c \rho^d V^e$

$$\frac{ML}{T^2} = L^a L^b \left[\frac{M}{LT}\right]^c \left[\frac{M}{L^3}\right]^d \left[\frac{L}{T}\right]^e$$

$$\Rightarrow \frac{ML}{T^2} = \frac{L^{a+b-c-3d+e} M^{c+d}}{T^{c+e}}$$

$L: a+b-c-3d+e = 1$

$M: c+d = 1$

$T: c+e = 2$

$\Rightarrow$ $e = 2-c$, $d = 1-c$, $a = 2-b-c$

$$D = w^{2-b-c} h^b \mu^c \rho^{1-c} V^{2-c}$$

$$D = w^2 \rho V^2 \left(\frac{h}{w}\right)^b \left(\frac{\mu}{w\rho V}\right)^c$$

$$\underbrace{\frac{D}{w^2 \rho V^2}}_{\pi_1} = \underbrace{\left(\frac{h}{w}\right)}_{\pi_2}^b \underbrace{\left(\frac{\mu}{w\rho V}\right)}_{\pi_3}^c$$

2. Given the following wind profile in a turbulent boundary layer flow over rough surface:

z (m)	U (m/s)
0.95	3
3.0	4
9.5	5
30.0	6

$\ln Z$

-0.0513

1.0986

2.25129

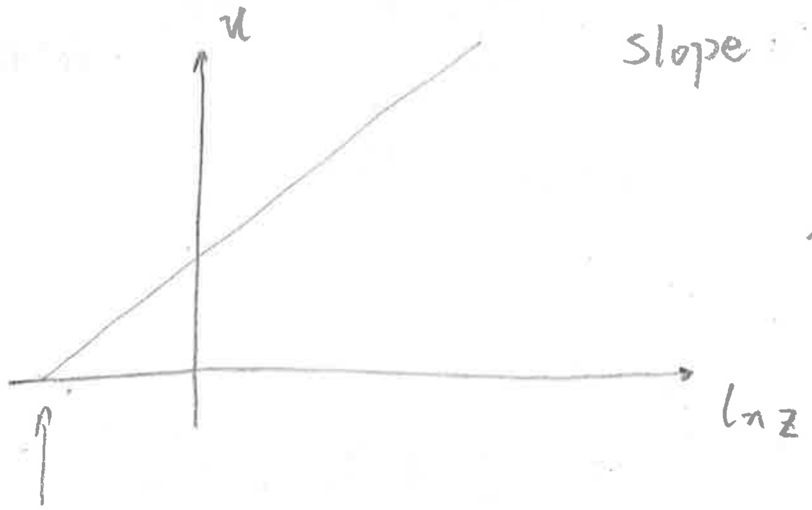
3.4012

use Log law of rough surface

$$\left(\frac{u}{u^*}\right) = \frac{1}{k} \ln\left(\frac{z}{z_0}\right)$$

Find the value of: a) surface roughness (z_o) and b) friction velocity (u_*).

$$\frac{u}{u^*} = \frac{1}{k} \ln \frac{z}{z_0} \Rightarrow u = \frac{u_*}{k} (\ln z - \ln z_0)$$



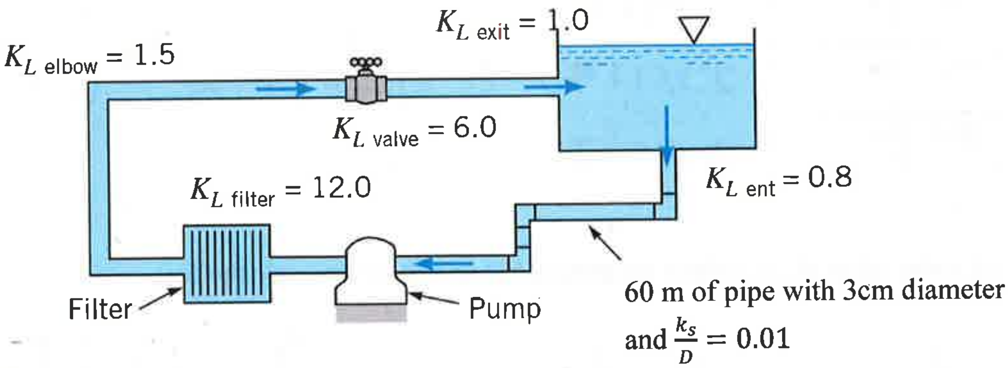
slope: $\frac{u_*}{k} \approx 0.87$

$k = 0.4$ (Von-karman constant)

$u_* \approx 0.35$ m/s

$\ln z_0 \approx -3.5$ $\quad z_0 \approx 0.03$ m

3. Water is circulated from a large tank, through a filter, and back to the tank as shown below. The power added to the power by the pump is 270 W. Determine the discharge through the filter. Neglect other component head losses which are not shown in the below figure.



Energy eq.

$$z_1 + \alpha\frac{V_1^2}{2g} + \frac{P_1}{\rho g} + h_p = z_2 + \alpha\frac{V_2^2}{2g} + \frac{P_2}{\rho g} + h_L$$

$$P_{pump} = \dot{m} g h_p = \rho\pi\frac{D^2}{4} V g h_p = 270\,W$$

$$h_p = \frac{4P_{pump}}{\rho\pi D^2 g V}$$

!

$$\sum K_L = K_{L\,ent} + 5\times K_{L\,elbow} + K_{L\,filter} + K_{L\,valve} + K_{L\,exit} = 27.3$$

$$\boxed{h_L = h_p} \Rightarrow \frac{4P_{pump}}{\rho\pi D^2 g V} = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)$$

$$V = \left(\frac{270\times 4\times 2}{1000\times 3.14\times 0.0009\,(f\times 2000 + 27.3)}\right)^{1/3}$$

Use Moody diagram. initial guess $f = 0.04$

$V = 1.93\,m/s$, $Re = 3.2\times 10^4 \Rightarrow$ new $f = 0.041$

$V = 1.91\,m/s$, $Re = 3.18\times 10^4$ 8 $\Rightarrow$ converge

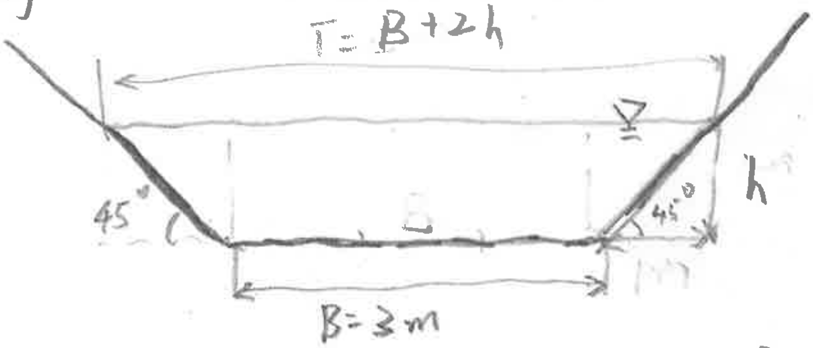
$Q = 1.36\times 10^{-3}\ m^3/s$

4. Consider a trapezoidal channel which bottom is 3.0m wide and the side slopes are 1.0m vertical to 1.0m horizontal.

A) If the discharge is 10 m^3/s and the flow depth is 1m, is the flow subcritical or supercritical?

B) If the discharge is 20 m^3/s determine the critical depth.

a)



A) $F_r = \frac{V}{\sqrt{g D}}$, $V = \frac{Q}{A} = \frac{10}{4} = 2.5 m/s$

$D = \frac{A}{T} = \frac{4}{5} = 0.8 m$

$F_r = \frac{2.5}{\sqrt{9.8 \times 0.8}} \approx 0.893 < 1$

The flow is sub critical

B) At the critical point:

$\frac{A_c^3}{T_c} = \frac{Q^2}{g}$, $A_c = \frac{(B + B + 2h_c) h_c}{2} = h_c(B + h_c)$

$T = B + 2h_c$

$\frac{h_c^3 (B + h_c)^3}{B + 2h_c} = \frac{Q^2}{g} = 40.8 m^2$

$h_c = \frac{(40.8 (B + 2h_c))^{1/3}}{B + h_c}$ ⇒ iterative solving

initial guess

$h_c = 1$, RHS = 1.47

$h_c = 1.47$, RHS = 1.395

$h_c = 1.395$, RHS = 1.406

$h_c = 1.406$, RHS = 1.405

Converge

$h_c = 1.405 m$